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V. L. RVACHEV'S QUASI-GREEN'S FUNCTIONS METHOD IN THE THEORY OF HEAT CONDUCTION

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We present a generalization of V. L. Rvachev's method of quasi-Green's functions in connection with the solution of mixed problems for the heat-conduction equation in noncylindrical domains.

Let Ω be a domain in a space of n + 1 dimensions (n = 2, 3), the boundary $\partial\Omega(S_{t_0} + S_{t'} + S_B)$ of which is represented by the normalized equation $\omega(P, t) = 0$, where P is a point with the coordinates $(x_1, x_2, \ldots, x_n)$. We assume that $\omega(P, t)$ is twice continuously differentiable with respect to the spatial coordinates and once continuously differentiable with respect to t; moreover, $\omega(P, t) > 0$ for all $(P, t) \in \Omega/\partial\Omega$ [1].

In the domain Ω we consider the problem of finding a solution of the heat-conduction equation

$$Lu = f\left(L = \Delta - \frac{1}{a^2}\frac{\partial}{\partial t}\right), \tag{1}$$

satisfying the conditions

$$u|_{S_{t_0}} = 0, \tag{2}$$

$$u|_{t=t_0} = 0. \tag{3}$$

It was shown in [2] that an arbitrary solution of the heat-conduction equation (1), twice continuously differentiable with respect to $(x_1, \ldots, x_n)$ and continuously differentiable with respect to t, can be represented in the following form:

$$u(P, t) = -a^2 \int_{t_0}^{t} \int \overset{(n-1)}{\underset{S_{t'}}{\cdots}} \int \left(v\frac{\partial u}{\partial n'} - u\frac{\partial v}{\partial n'}\right) dS' dt' + \\ + \int \overset{(n)}{\underset{G_{t_0}}{\cdots}} \int uv d\tau' + \int \overset{(n-1)}{\underset{S_B}{\cdots}} \int uv\cos(n^*, \mathbf{t}) dS' - a^2 \int_{t_0}^{t} \int \overset{(n)}{\underset{G_{t'}}{\cdots}} \int v Lu\, d\tau' dt', \tag{4}$$

where

$$v = \delta(P, P', t, t') = \left(\frac{1}{2a\sqrt{\pi(t-t')}}\right)^n \exp\left(-\frac{r^2}{4a^2(t-t')}\right); \tag{5}$$

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δ is the fundamental solution of the heat-conduction equation.

Green's second formula for Eq. (1) may be written

$$\int_{t_0}^{t}\int \underset{G_{t'}}{\overset{(n)}{\cdots}} \int (gLu - uMg)\,d\tau' dt' = -\int_{t_0}^{t}\int \underset{S_{t'}}{\overset{(n-1)}{\cdots}} \int \left(g\frac{\partial u}{\partial n'} - u\frac{\partial g}{\partial n'}\right) dS' dt' + \int \underset{G_{t_0}}{\overset{(n)}{\cdots}} \int \frac{ug}{a^2}\,d\tau' + + \int \underset{S_B}{\overset{(n-1)}{\cdots}} \int \frac{ug}{a^2} \cos(n^*,\ \mathbf{t})\,dS' - \int \underset{G_{t'}}{\overset{(n)}{\cdots}} \int \frac{ug}{a^2}\,d\tau', \tag{6}$$

where $M = \Delta + 1/a^2\ \partial/\partial t$ is the operator adjoint to the heat-conduction operator L.

It follows from Eqs. (4) and (6) that

$$u(P,\ t) = -a^2\int_{t_0}^{t}\int \underset{G_{t'}}{\overset{(n)}{\cdots}} \int (gLu - uMg)\,d\tau' dt' - -a^2\int_{t_0}^{t}\int \underset{S_{t'}}{\overset{(n-1)}{\cdots}} \int \left(g\frac{\partial u}{\partial n'} - u\frac{\partial g}{\partial n'}\right) dS' dt' + \int \underset{G_{t_0}}{\overset{(n)}{\cdots}} \int ug\,d\tau' - \int \underset{G_{t}}{\overset{(n)}{\cdots}} \int ug\,d\tau' + + \int \underset{S_B}{\overset{(n-1)}{\cdots}} \int ug\cos(n^*,\ \mathbf{t})\,dS' - a^2\int_{t_0}^{t}\int \underset{S_{t'}}{\overset{(n-1)}{\cdots}} \int \left(\delta\frac{\partial u}{\partial n'} - u\frac{\partial \delta}{\partial n'}\right) dS' dt' + + \int \underset{G_{t_0}}{\overset{(n)}{\cdots}} \int u\delta\,d\tau' + \int \underset{S_B}{\overset{(n-1)}{\cdots}} \int u\delta\cos(n^*,\ \mathbf{t})\,dS' - a^2\int_{t_0}^{t}\int \underset{G_{t'}}{\overset{(n)}{\cdots}} \int \delta Lu\,d\tau' dt'.$$

We rewrite the last relation in the form

$$u(P,\ t) = -a^2\int_{t_0}^{t}\int \underset{G_{t'}}{\overset{(n)}{\cdots}} \int (g+\delta) Lu\,d\tau' dt' + a^2\int_{t_0}^{t}\int \underset{G_{t'}}{\overset{(n)}{\cdots}} \int uMg\,d\tau' dt' + \int \underset{G_{t_0}}{\overset{(n)}{\cdots}} \int u(g+\delta)\,d\tau' - -a^2\int_{t_0}^{t}\int \underset{S_{t'}}{\overset{(n-1)}{\cdots}} \int \left[(g+\delta)\frac{\partial u}{\partial n'} - u\frac{\partial}{\partial n'}(g+\delta)\right] dS' dt' + \int \underset{S_B}{\overset{(n-1)}{\cdots}} \int u(g+\delta)\cos(n^*,\ \mathbf{t})\,dS' - \int \underset{G_{t}}{\overset{(n)}{\cdots}} \int ug\,dS'. \tag{7}$$

As g we choose the function

$$g(P,\ P',\ t,\ t') = \left(\frac{1}{2a\sqrt{\pi(t-t')}}\right)^n \exp\left(-\frac{r^2 + 4\omega(P,\ t)\,\omega(P',\ t')}{4a^2(t-t')}\right). \tag{8}$$

By virtue of the relations (5) and (8) we have

$$g + \delta|_{t=t',\,P\neq P'} = 0,\quad g + \delta|_{S_B} = 0. \tag{9}$$

We then obtain from the relation (7), taking into account the relations (1)-(3) and (9),

$$u(P,\ t) = -a^2\int_{t_0}^{t}\int \underset{G_{t'}}{\overset{(n)}{\cdots}} \int (g+\delta) f\,d\tau' dt' + a^2\int_{t_0}^{t}\int \underset{G_{t'}}{\overset{(n)}{\cdots}} \int u(P',\ t') K(P,\ P',\ t,\ t')\,d\tau' dt', \tag{10}$$

where

$$K(P,\ P',\ t,\ t') = \frac{\omega(P,\ t)}{a^2(t-t')\,(2a\sqrt{\pi(t-t')})^n} \exp\left(-\frac{r^2 + 4\omega(P,\ t)\,\omega(P',\ t')}{4a^2(t-t')}\right) \times \times \left[\frac{\omega(P,\ t)(\nabla\omega(P',\ t'))^2 - \mathbf{r}\cdot\nabla\omega(P,\ t) - \omega(P',\ t')}{a^2(t-t')} - M\omega(P',\ t')\right].$$

Ordinary considerations [1] readily serve to establish the continuity of K(P, P', t, t') in the domain Ω; Eq. (10), therefore, represents a Fredholm integral equation of the second kind for determining the solution of the initial problem (1)-(3). Established numerical methods [3] may be employed to solve Eq, (10).

This version of V. L. Rvachev's quasi-Green's function method may also be immediately generalized to the case of nonhomogeneous initial and boundary conditions.

NOTATION

G, a finite domain of a three- or two-dimensional space; S, the piecewise-smooth boundary of the domain G; $G_{t'}$, (G_{t_0}), the spatial domain for t' = const (t_0 = const) with the boundary $S_{t'}$(S_{t_0}); t, time; dS', area element of the boundary G; dτ', volume element of the domain G; $\mathbf{n}' = \mathbf{n}'(P', t')$, inner normal to the boundary $S_{t'}$ at the point P'; $r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$ in the three-dimensional case, $r = \sqrt{(x-x')^2 + (y-y')^2}$ in the two-dimensional case; $\mathbf{n}^*$, inner normal to the boundary of the domain Ω; $\mathbf{t}$, unit vector along the Ot axis.

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